1. The proof is by induction on .

Basic Step: We have or , so the claim is true for .

Induction Hypothesis: Assume that the claim is true from some positive integer . That is, . Multiply each side by 2, so we have . That can also be written as. Looking at the right-hand side, . Since is a positive integer, , and then is always true.

Therefore, by induction, the statement is true for all positive integers.

1. The proof is by induction on .

Basic Step: We have , so the claim is true for .

Induction Hypothesis: Assume that the claim is true for some positive integer . That is . Add to both sides to get . The left-hand side can be written as .

Therefore, by induction, the statement is true for all positive integers.

1. The proof is by induction on.

Basic Step: We have , which is a multiple of 3, so the claim is true for .

Induction Hypothesis: Assume that the claim is true for some natural number . That is, . By the definition of divides, this means that for some integer . Multiply to both sides of the equation to get . Distribute the 4 of the left-hand side to get . Add to both sides to get . Factor out the 3 from the right-hand side and simplify the left-hand side to get . So there is an integer such that , namely , so .

Therefore, by induction, the statement is true for all natural numbers.

1. The proof is by induction on.

Basic Step: We have or , so the claim is true for .

Induction Hypothesis: Assume that the claim is true for some positive integer . That is, Add to both sides of the equation to get , . The left-hand side can also be written as . Factoring out the in the numerator gives us . Multiplying out gives us , or which can be written as .

Therefore, by induction, the statement is true for all positive integers.

1. The proof is by induction on .

Basic Step: We have , or , so the claim is true for n = 0.

Induction Hypothesis: Assume that the claim is true for some natural number . That is, . Adding to both sides gives us . By the definition of the Fibonacci sequence, . can be written as . So

Therefore, by induction, the statement is true for all natural numbers.

1. 1. No, the relation ⊗ is not reflexive. For example, when , and , so , but , so is not related to itself.
   2. No, the relation ⊗ is not irreflexive. For example, when , , so x⊗x.
   3. Yes, the relation ⊗ is symmetric. Suppose , that means . Since , then . Thus .
   4. No, the relation ⊗ is not antisymmetric. For example, when and , then , so , and , but
   5. No, the relation ⊗ is not transitive. For example, let , , and . and , so and , but , so is not related to .